Using Categorical Data Analysis to Estimate Population Change in EU25: Absolute Numbers and Crude Rates

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Abstract — Statistics on population change and the structure of population are increasingly used to support policymaking and to provide the opportunity to monitor demographic behaviour within political, economic, social and cultural contexts. Specifically, this concerns demographic developments that focus on a likely reduction in the relative importance of the working age population and a corresponding increase in the number of older persons. These statistics may be used to support a range of different analyses, including studies relating to population ageing and its effects on the sustainability of public finance and welfare, the evaluation of fertility as a background for family policies, or the economic and social impact of demographic change. This research aims to highlight the population change in twenty-five countries of the European Union. We consider the use of categorical data analysis to estimate the population change in EU25: absolute numbers and crude rates from 2003-2017. The data used in this study are from the Eurostat/World population prospects and estimated on actual base year from. Since the main focus is to have a better understanding of the population change in EU25, the analysis of association table (ANOAS) is given in order to ascertain the percentage of the data which is covered by each model. We find and estimate the association model with the best fit and in conclusion we find out that the Row-Column Effects Association Model (RC) of the multivariate model (M=4) has the best fit among all - covering a total of 99.9% of the data observed.

Index Terms — Association models, Log-linear and non-linear models, population and EU25.

I. INTRODUCTION

Population change or population growth in a given year is the difference between the population size on 1 January of the given year and the corresponding level from 1 January of the previous year. It consists of two components: natural change and net migration plus statistical adjustment. Natural population change is the difference between the number of live births and the number of deaths. If natural change is positive, then it is often referred to as a natural increase. Net migration is the difference between the number of immigrants and the number of emigrants. The rate may be

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positive or negative. The growth rate is a factor in determining how great a burden would be imposed on a country by the changing needs of its people for infrastructure (e.g., schools, hospitals, housing, roads), resources (e.g., food, water, electricity), and jobs. Rapid population growth can be seen as a threat to a nation as well as threatening by neighboring countries.

Europe's ageing society and its relatively static number of inhabitants may be contrasted against a rapid expansion in the world's population, driven largely by population growth in developing countries. However, the demographic challenge that the EU-27 is confronted with is by no means unique. Most developed, and also some emerging economies, will undergo considerable changes in their demographic composition in the next five decades. Shrinking working age populations, a higher proportion of elderly persons, and increasing old-age dependency rates suggest that there will be a considerable burden to provide for social expenditure related to population ageing (pensions, healthcare, institutional care). The challenges associated with an ageing society are likely to be even more acute in countries such as Japan and South Korea where the old-age dependency ratio will rise rapidly and to a very high level, while the share of older persons in the total population is also expected to increase at a rapid pace in China [1].

A. Historical background

The European Union (EU) is a confederation of twentyeight nations. It was established in 1992 by the Treaty of the European Union (Maastricht Treaty) and it is the de facto successor to the European Communities (ECSC, EEC, EURATOM) of the six member states established in 1951, 1957 and 1965 respectively. Since then, new enlargements have increased the number of its member States and its competencies have been extended. It is an open process of European integration and it is one of the largest economic and political entities in the world [2]. Luxembourg City, together with Brussels and Strasbourg, is one of the three official capitals of the European Union and the seat of the European Court of Justice – the highest judicial authority in the EU. The European Union is a single market with a common commercial, agricultural, fisheries and regional policy to help the poorer regions. It introduced a single currency, the Euro, adopted by 15 Member States.

The European Union was also formally established when the Maastricht Treaty came into force on 1 November 1993. In 1995 Austria, Sweden and Finland entered the newly established Union. The next treaty was signed in Amsterdam in 1997. In 1999 the euro replaced the national currencies in 11 Member States, the so-called Eurozone. Greece joined the Euro in 2001. In 2004, ten new countries (eight of which were formerly communist countries) joined the European Union. In addition, Romania and Bulgaria joined the EU in 2007, while Croatia joined in 2013. Slovenia also joined the Eurozone in 2007. In 2008 Cyprus and Malta adopted the euro respectively [3].

B. Europe Population from 2014–2017 (in million inhabitants)

Europe which is about 11% of the world's population is the third-most populous continent after Asia and Africa with a population of 739 million. The population of the EU27 was 507.24 million in 2014, 508.52 million in 2015, 510.18 million in 2016 and 511.37 million in 2017. Europe is bordered by the Arctic Ocean to the north, the Atlantic Ocean to the west, the Mediterranean Sea to the south, and the Black Sea and connected waterways to the southeast. Europe is a peninsula that makes up the western fifth of the Eurasian landmass. Its maritime borders are made up of the Arctic Ocean to the north, the Atlantic Ocean to the west, and the Mediterranean, Black, and Caspian Seas to the south. The southern regions are more mountainous, while moving north the terrain descends from the high Alps, Pyrenees, and Carpathians, through hilly uplands, into broad, low northern plains, which are vast in the east. This extended lowland is known as the Great European Plain, and at its heart lies the North German Plain. An arc of uplands also exists along the north-western seaboard, which begins in the western parts of the islands of Britain and Ireland, and then continues along the mountainous, fjord-cut spine of Norway.

C. European Population Growth

Europe's population growth rate is approximated 0.212%. The average annual percent change in the population, resulting from a surplus or deficit of births over deaths and the balance of migrants entering and leaving a country. The rate may be positive or negative. The growth rate is a factor in determining how great a burden would be imposed on a country by the changing needs of its people for infrastructure, resources, and jobs. Rapid population growth can be seen as threatening by neighboring countries.

D. European Union Member States

The European Union is described as "a family of democratic European countries". On 23 July 1957 six founding members formed the European Coal and Steel Community, which was the first stage of today's EU. Twenty-one countries have since been introduced into successive waves of enlargement. The EU17 consists of Belgium, Germany, Estonia, Ireland, Greece, Spain, France, Italy, Cyprus, Luxembourg, Malta, the Netherlands, Austria, Portugal, Slovenia, Slovakia and Finland, while the EU28 includes Belgium, Bulgaria, the Czech Republic, Denmark, Germany, Estonia, Ireland, Greece, Spain, France, Italy, Cyprus, Latvia, Lithuania, Luxembourg, Hungary, Croatia, Malta, the Netherlands, Austria, Poland, Portugal, Romania, Slovenia, Slovakia, Finland, Sweden and the United Kingdom.

TABLE 1: EUROPEAN UNION MEMBER STATES [4]

Year	Enlargement History	Total
1952/1958	Belgium Germany France Italy Luxemburg Holland	6
1973 ²	Denmark Ireland United Kingdom	9
1981	Greece	10
1986	Spain Portugal	12
1995 ²	Austria Finland Sweden	15
2004	Czech Republic Cyprus Estonia Hungary Latvia Lithuania Malta Poland Slovakia Slovenia	25
2007	Bulgaria Romania	27
2013	Croatia ⁴	28

Notes:

- ¹ On 3 October 1990, East Germany joined with West Germany and its territory automatically became part of the EU.
- ² In 1973, as in 1995, Norway, despite the fact that it signed the respective Accession Treaties, did not endorse them because of the negative referendum of the Norwegian citizens.
- ³ Greenland, a self-governing territory under Danish sovereignty since 1979, left the European Communities in 1985 following a referendum.
- ⁴ The population of Croatia in 2013 was 4,475,611 inhabitants and became the 28th member of the EU on July 1, 2013. In April 2009 it became a member of NATO.

Any country willing to join the European Union must adhere to the economic and political terms, known as the Copenhagen Criteria (following the Copenhagen Summit in June 1993). This basically requires a secular, democratic government, the rule and respect of law and corresponding social and institutional freedoms. Under the EU Treaty, each Member State and the European Parliament must agree unanimously on any enlargement.

E. Candidate countries (En route to EU membership)

There are currently six candidate countries to join the EU. These are:

- Albania;
- Republic of North Macedonia;
- Iceland;
- Montenegro;
- Serbia;
- Turkey.

In addition, the Balkan countries Bosnia and Herzegovina and Kosovo are recognized as potential candidates.

F. Demographics Data Population

The most populous country in the EU is Germany with 82,2 million inhabitants. The top five are: The United Kingdom with (65.65 million), France (64.1 million), Italy (59.29 million) and Spain (46.4). Greece ranks 10th with a population of 11.1 million according to the official census of 2017. The country with the smallest population in the EU is Malta with only 0.4 (432089) million Residents followed by Luxemburg with 0.5 (590321) million of which 50.9% are Luxembourgers while 49.1% are foreign citizens. Cyprus is

followed after Luxembourg with 1.19 million. The largest city in the EU is London, with the population of the entire metropolitan complex according to 2011 census, 8 173 941. The metro population in 2018 estimated the population of London to be 8 787 892.

In 2011, natural increase (the positive difference between live births and deaths) added 0.4 million (32 %) to the population growth in the EU-27. Some 68 % of the EU-27's population growth came from net migration plus statistical adjustment, which continued to be the main determinant of population growth, contributing 0.9 million in 2011.

The contribution of net migration plus statistical adjustment to the total population change in the EU-27 has been greater than that of natural change since 2002. The share of net migration plus statistical adjustment in total population growth peaked, in relative terms, in 2003 (95 % of total change). Since this date, the contribution of net migration plus statistical adjustment decreased somewhat. Thus, the share of natural change in total population growth followed an upward development up to 2009 (37%) and declined again over the most recent years (from 2010 onwards). In continuation, Table 2 below shows the 10 largest cities in the EU.

TABLE 2: TOP 10 LARGEST CITIES IN THE EUROPEAN UNION - (ESTIMATE

2017) [3]

A/A	Country	Town Planning Complex	Population
1	France	Paris	122741346
2	United Kingdom	London	8 787892
3	Germany	Berlin	7 245576
4	Spain	Madrid	6514.620
5	Spain	Barcelona	64393081
6	Germany	Essen – Bochum- Dortmund	5 126013
7	Italy	Milan	4851.565
8	Italy	Rome	4039,813
9	Greece	Athens	3750.124
10	Poland	Warsaw	1 781740

G. European population compared with world population

By 1 July 2017 the world's population had reached 7 547 859 (Seven billion, five hundred and forty-seven million, eight hundred and fifty nine thousand) inhabitants, according to United Nations' (UN's) World Population Prospects, 2010 revision. Most population scenarios project that population numbers will continue to grow in the coming decades. Asia accounted for the majority of the world's population (just over 60 % in 2012) with 4.250 billion inhabitants, while Africa was the next most populous continent with 1.070 billion inhabitants, or 15.2 % of the global total. In comparison, there were 504 million inhabitants in the EU-27 in 2012, accounting for just over 7 % of the world's population.

Global population doubled between 1960 and 2000. From the turn of the millennium to 2012, the world's population grew by a further 929 million persons. The overall increase in global population between 1960 and 2017 can be largely attributed to growth in Asia, Africa and Latin America

The latest UN population projections that the pace at which the world's population is expanding will slow somewhat in the coming decades; nevertheless, the total number of inhabitants is projected to reach more than 9 550 944 891 by 2050. According to these projections (the medium variant), the world's population will also be relatively older (in other words, with a higher median age) in 2050. At the same time, according to Eurostat's population projections, the EU-27's population is predicted to grow at a slower rate before peaking in 2040-45, contracting thereafter.

H. World Population Prospects 2017

The 2017 world population of 7.6 billion is expected to reach 8.6 billion in 2030, 9.8 billion in 2050 and 11.2 billion in 2100, according to a new United Nations report being launched today. With roughly 83 million people being added to the world's population every year, the upward trend in population size is expected to continue, even assuming that fertility levels will continue to decline [1].

TABLE 3: CONTINENTAL POPULATION

Asia	4 140 000 000
Africa	995 000 000
Europe	739 000 000
North America	529 000 000
South America	386 000 000
Australia	36 000 000

I. Shifts in country population rankings / Population 2050 (estimate)

The new projections include some notable findings at the country level. China (with 1.4 billion inhabitants) and India (1.3 billion inhabitants) remain the two most populous countries, comprising 19% and 18% of the total global population. By 2024, or in roughly seven years, the population of India is expected to surpass that of China.

TABLE 4: COUNTRY POPULATION RANKINGS 2050 (ESTIMATE)

	Total	9 550 944 891
10	Philippines	157 117 506
9	Ethiopia	187 572 656
8	Bangladesh	201 947 716
7	Brazil	231 120.024
6	Pakistan	271 081 825
5	Indonesia	321 377 092
4	USA	400 853.042
3	Nigeria	440 355 062
2	China	1 384 976 976
1	India	1 620 050 849

Among the ten largest countries worldwide, Nigeria is growing the most rapidly as shown in Table 4 above. Consequently, the population of Nigeria, currently the world's 7th largest, is projected to surpass that of the United States and become the third largest country in the world shortly before 2050.

J. Large movements of refugees and other migrants

There continue to be large movements of migrants between regions, often from low- and middle-income countries toward high-income countries. The volume of the net inflow of migrants to high-income countries in 2010-2015 was (3.2 million per year). Although international migration at or around current levels will be insufficient to compensate fully for the expected loss of population tied to low levels of fertility, especially in the European region, the movement of people between countries can help attenuate some of the adverse consequences of population ageing.

The Syrian refugee crisis has had a major impact on levels and patterns of international migration in recent years, affecting several countries. Generally, the net outflow from the Syrian Arab Republic was 4.2 million persons from 2010-2017. Most of these refugees migrated to Syria's neighboring countries, contributing to a substantial increase in the net inflow of migrants especially to Turkey, Greece, Lebanon and Jordan.

II. STATISTICAL DATA AND PROCESSING OF THE RESULTS

The data used for this study was obtained from the Eurostat/World Population Prospects, the statistical office of the European Union, calculated on an annual basis. The processing of observations (data) consists of the population change in the EU25: absolute numbers and crude rates. Below in Table 5 are the lists of data under our study for the subsequent years from 2003-2017.

TABLE 5: POPULATION CHANGE IN EU27: ABSOLUTE NUMBERS AND CRUDE RATES [5]

TABLE 5.1 OF CLATION CHANGE IN EC27. ABSOLUTE NUMBERS AND CRODE RATES [5]									
geo time	2003	2004	2005	2006	2007	2008	2009	2010	2011
Belgium	10 004 500	10045200	10084500	10115600	10136800	10156600	10181300	10203100	10226500
Czech Republic	10 308 600	10319100	10329900	10333600	10327300	10315200	10304100	10294400	10283900
Denmark	5 154 300	5171400	5188600	5206200	5233400	5263100	5285000	5304300	5321800
Germany	80 013 900	80624600	81156400	81438400	81678100	81914900	82034800	82047200	82100300
Estonia	1561300	1533100	1494200	1462600	1436700	1415600	1399600	1386200	1375700
Ireland	3534300	3558500	3576300	3590400	3608900	3637600	3674300	3712900	3755000
Greece	10256300	10369900	10465600	10553100	10634400	10709200	10776500	10834900	10882600
Spain	38939100	39067700	39189400	39295000	39387000	39478200	39582400	39721100	39926300
France	58459145	58745390	58995125	59209730	59418718	59624342	59830635	60046709	60320085
Italy	56758700	56797700	56832800	56844700	56846100	56862800	56893800	56911000	56921600
Cyprus	595100	611200	626100	639200	650900	661300	670800	679100	686700
Latvia	2650600	2614400	2563300	2520800	2485100	2457200	2432900	2410000	2390500
Lithuania	3704200	3700100	3682600	3657200	3629100	3601600	3575200	3549400	3524300
Luxembourg	387000	392200	397500	403000	408700	414300	419500	424700	430500
Hungary	10373400	10369300	10357500	10343400	10329000	10311200	10290500	10266600	10237500
Malta	357700	361300	364700	368000	370400	372600	375300	377500	379400
Netherlands	15069800	15184200	15290400	15382900	15459000	15530500	15610700	15707200	15812100
Austria	7754900	7840700	7905600	7936100	7948300	7959000	7968100	7976800	7992400
Poland	38246200	38363700	38461400	38542700	38595000	38624400	38649700	38663500	38660300
Portugal	9967900	9970000	9982600	10004100	10030400	10057900	10091100	10129300	10172000
Slovenia	1999400	1996500	1991800	1989500	1989900	1988700	1986000	1981600	1983100
Slovakia	5303300	5305100	5325400	5346400	5362000	5373400	5383300	5390600	5396100
Finland	5013800	5042000	5066500	5088400	5107800	5124600	5139800	5153500	5165500
Sweden	8617400	8668100	8718600	8780800	8827000	8841000	8846100	8851000	8857900
United Kingdom	59644900	59950400	60418600	60035700	61329000	6182900	62267000	62767100	63182500

geo time	2012	2013	2014	2015	2016	2017
Belgium	10 251 300	10 286 600	10 332 800	10 376 100	10 421 137	10 478 617
Czech Republic	10 272 300	10 236 500	10 204 900	10 207 400	10 216 016	10 235 828
Denmark	5339600	5358800	5376000	5390600	5404523	5419432
Germany	82211500	82349900	82488500	82534200	82516260	82469422
Estonia	1369600	1364100	1358600	1353600	1349290	1346097
Ireland	3805400	3866500	3931800	3995700	4068453	4159096
Greece	10917500	10950000	10987600	11023600	11061701	11103978
Spain	40263200	40720500	41314000	42004500	42691689	43398143
France	60667362	61044483	61425570	61799630	62251286	62702371
Italy	56948600	56980700	57157400	57604700	58175310	58607043
Cyprus	694000	701500	710300	722800	739771	757795
Latvia	2 373 000	2 355 100	2 338 700	2 325 400	2 312 819	2 300 512
Lithuania	3 499 600	3 481 300	3 469 100	3 454 300	3 435 591	3 414 304
Luxembourg	436300	441600	446200	450000	453300	457250
Hungary	10211000	10187600	10158700	10129600	10107146	10087065
Malta	385800	393000	396000	398600	401268	403507
Netherlands	15925600	16046200	16149000	16225300	16281779	16319868
Austria	8011600	8043000	8083700	8121200	8173323	8236225
Poland	38453800	38248100	38230400	38204600	38182222	38165445
Portugal	10225900	10293000	10368400	10441100	10501970	10549424
Slovenia	1989000	1992100	1994500	1995700	1997012	2000474
Slovakia	5388800	5378900	5379100	5379700	5382438	5387001
Finland	5176200	5188000	5200600	5213000	5228172	5246096
Sweden	8872100	8896000	8925000	8958300	8993531	9029572
United Kingdom	63792500	64118700	64627700	65118800	65389864	65648054

With the help of the Categorical Data Analysis program (CDAS), we were able to ascertain the results of our data for the population change in the EU25: absolute numbers and crude rates from 2003-2017. Table 6 below shows the values (prices) of all the models to be estimated. The statistics used to compare and contrast the association (correlation) of the models are X² (Pearson) Chi-Square and the Likelihood-Ratio Chi-Square G²-probability ratio [6]. In continuation, the following results are shown below:

TABLE 6: DATA RESULTS								
Models	X ² (Pearson)Chi- Square	Likelihood-Ratio Chi-Square G ²	Degrees of Freedom	Index of Dissimilarity	Maximum Deviation			
0	123796.93577	123599.24741	80	0.00265	0.00000003			
U	103522.90731	103376.02003	79	0.00236	0.00061689			
R	103394.71974	103242.19437	70	0.00236	0.00070555			
C	20439.20893	20437.38281	72	0.00104	0.00001520			
R+C	20306.16632	20301.82262	63	0.00103	0.00008217			
RC	14672.60555	14675.23959	63	0.00092	0.84482370			

A. Association Model

In the categorical data analysis system, we apply the methodology by considering six of the most commonly used association models. These are:

1) The model of Independence or null association model which is also symbolized by (O) and holds that there is no relationship between the variables. The log-linear model is:

$$\log(F_{ij}) = \lambda + \lambda_{A(i)} + \lambda_{B(j)},$$

where log denotes the natural logarithm,

 F_{ij} the expected frequencies under the independence model, $\lambda_{A(i)}$ are the rows main effects,

 $\lambda_{B(i)}$ are the columns main effects [7].

2) The Uniform association model, which is symbolized by (U) in log-linear form is:

$$\log(F_{ij}) = \lambda + \lambda_{A(i)} + \lambda_{B(j)} + \varphi \chi_i y_i$$

where φ is a single parameter for interaction, $\chi_i y_j$ are the scores for the row and column variables (i =1,...,I, j=1,...,J) respectively.

3) The row effects model (R) where linear-by-linear interaction holds that [8],

$$\log(F_{ij}) = \lambda + \lambda_{A(i)} + \lambda_{B(j)} + \varphi \mu_I y_j$$

where y_i are fixed scores for the column variable (j = 1,...,J), μ_i are unknown scores for the row variable (i =1,...,I).

4) The column effects model (C) is the same as the R model with a change in subscripts:

$$\log(F_{ij}) = \lambda + \lambda_{A(i)} + \lambda_{B(j)} + \varphi v_j x_I$$

where x_i are fixed scores for the row variable (i =1,...,I), v_i are unknown scores for the column variable (i = 1,...,J).

5) The model that allows both row and column effects in additive form is called the R+C model [9]. The logfrequency version of the above model is

$$\log(F_{ij}) = \lambda + \lambda_{A(i)} + \lambda_{B(j)} + \sum_{k=1}^{I-1} \beta_k y_j Z_{A(k)} + \sum_{k=1}^{J-1} y_k x_j Z_{B(k)},$$

where χ_i, y_i are the scores (as defined earlier), $Z_{A(i)}$, $Z_{B(i)}$ denote to variables (dummy variables) for the row and column levels respectively.

6) The model, instead of additive row and column effects on the local odds ratios, has multiplicative effects called the R*C model or model II [10]. The log-multiplicative model

$$\log(F_{ij}) = \lambda + \lambda_{A(i)} + \lambda_{B(i)} + \varphi \mu_I \nu_i$$

where the row score parameters μ_I and column score parameters v_i are not known but are estimated in the data.

We aim at finding out the model that has the best fit among the 6 models which we are examining, i.e., population change in the EU25: absolute numbers and crude rates from 2003-2017. For this reason, first, we are going to examine the Index of Dissimilarity (L2), which shows that, the lesser the number, the more our model will give the best fit to match the data under consideration.

We analyze the six association model describe above, with the help of the categorical data analysis statistical program [11]. We used the Pearson chi-squared (X²) statistics, the likelihood-ratio chi-square (G²) statistics and the index of dissimilarity which is equated by:

$$D = \sum_{I_j} \frac{\left| \frac{f_{ij}}{n} - \frac{F_{ij}}{n} \right|}{2} \tag{1}$$

 f_{ij} are the observed frequencies,

 F_{ij} are the expected frequencies (under the model).

Additionally, we have the following results as shown in the table below:

TABLE 7: INDEX OF DISSIMILARITY							
	Models	Index of Dissimilarity (D)					
1.	Null Association-Independence Model (O)	0. 00265					
2.	Uniform Association Model (U)	0. 00236					
3.	Row-Effects Association Model (R)	0. 00236					
4.	Column-Effects Association Model (C)	0. 00104					
5.	Row+Column Effects Association Model (R+C)	0. 00103					
6.	Row Column Effects Association Model (R*C)	0. 00092					

At first sight in Table 7 above, it seems that the Row Column Effects of the Association Model (R*C) adjusted better to the percentage of the population change in the EU25for the years under our study (having the lowest index of dissimilarity) with D=0.00092.

Since we have models with similar lower ratio, we justify the model with the best fit to match both countries and years by calculating the Index BIC (Bayesian Information criterion). In statistics, the Bayesian Information Criterion (BIC) or Schwarz criterion is a Criterion for model selection among a finite set of models; the model with the lowest BIC is preferred. It is based, in part, on the likelihood function. When fitting models, it is possible to increase the likelihood by adding parameters but doing so may result in over-fitting. BIC attempt to resolve this problem by introducing a penalty term for the number of parameters in the model and gives the best solution [12].

The formula for this calculation is:

$$BIC = G^2 - (D.F.) \log (n) \tag{2}$$

Notations:

 G^2 = the likelihood-ratio chi-square statistics;

d.f. = degrees of freedom of the models;

n = the size of the sample (2240630715.0000);

Log(n) = Log(2240630715.0000) = 21.5300232.

When comparing a number of models, the model with the smallest index of BIC is assumed to be the best. So, we choose the models that have similar and lowest INDEX OF DISSIMILARITY out of the six models. More precisely, we will consider the 3rd, 5th and 6th models respectively.

Subsequently, the calculation is as follows:

 3^{rd} model: BIC = G^2 – (D.F.) Log (n) = 21197.91551 -(72*21.5300232 = 19647.7538

 5^{th} model: BIC = G^2 – (D.F.) Log (n) = 21036.38760– (63*21.5300232) = 19679.9961

 6^{th} model: BIC = G^2 – (D.F.) Log (n) = 15327.73587– (63*21.5300232) = 13971.3444

As we can see from the above calculations, the 6th model (Row Column-effects of the Association Model (RC)) finally accounts for the best fit from the results since it has the smallest index of BIC.

B. Analysis of the Association Models

In continuation, the association model passes through several tests to ascertain the accuracy, quality, or satisfactory fit of each model. Tests are done through the use of the likelihood-ratio chi-square (G²) statistics and the Pearson chi-squared (X^2) distribution. In the case of the X^2 distribution, the Statgraph programme will be of good help.

Initially, we observe that the likelihood-ratio chi-square statistic for the Independence model (O) which is G^2 =123960.12346 with 80 degrees of freedom. The 95% reference point chi-square distribution is 102.023. It has unacceptable fit because the X2 distribution is smaller than the likelihood-ratio chi-square statistic G².

Subsequently, the Uniform association model has G²=104091.90963 with 79 degrees of freedom. The 95% reference point chi-square distribution is 100.887. As it could be noticed, this statistic is not accepted and does not have a satisfactory fit since the X² distribution is much smaller than the likelihood-ratio chi-square statistic G².

Moreover, the statistic G² for the Row model (R) is reduced dramatically for 103931.40198 with 70 degrees of freedom. The 95% reference point chi-square distribution is 90.6892. The row model is also not accepted because the X^2 distribution is smaller than the likelihood-ratio chi-square statistic G².

The Column model (C) has $G^2=21197.91551$ with 72 degrees of freedom. The 95% reference point chi-square distribution is 92.9779 which show even the worst fit as we could observe that the X² distribution is very much smaller than the likelihood-ratio chi-square statistic G^2 .

The statistics of the model R+C, that takes into account the effects for both Countries and Years in additive form, is G²=21036.38760 with 63 degrees of freedom. The 95% reference point chi-square distribution is 82.6494 has equally unacceptable fit since the X² distribution is smaller than the likelihood-ratio chi-square statistic G².

Finally, the model RC, that is log multiplicative but not log-linear, the G² Statistics is 15327.73587 with 63 degrees of freedom. The 95% reference point chi-square distribution is 82.6494. Furthermore, the statistics is dramatically reduced just as the previous model because they have identical degrees of freedom but is shown to remain unacceptable fit because the X2 distribution is very much smaller than the likelihood-ratio chi-square statistic G².

By virtue of the index of dissimilarity of the models, the model R+C has the best fit. However, upon clear test to ascertain the accuracy, quality, or satisfactory fit of the model, we find out that it has a very poor fitting, making it unacceptable [13], [14].

Consequently, we have to find out the degree or level of effects on each model. To verify this, we will construct the analysis of association (ANOAS) table.

C. Analysis of Association Table (ANOAS)

The ANOAS table in table 6 below was given by [15]. In this table, the X² is divided so that it can be used as two factor analysis of variance by making use of the G² (0) statistics for the base (zero) independence model which measures the total deviation of the variables. In other words, we can find the percentage of the baseline chi-squared X² distribution, which have effects on each of the models on the phenomenon being studied.

TABLE 8: ASSOCIATION ANALYSIS

Models	Likelihood-G ²	Degrees of Freedom	Index of Dissimilarity
О	123960.12346	80	0.00266
U	104091.90963	79	0.00237
R	103931.40198	70	0.00237
C	21197.91551	72	0.00104
R+C	21036.38760	63	0.00103
RC	15327.73587	63	0.00093

The analysis of association table has the following differences of our models: O-U is the overall or total effects of the models, U-C are the column effects model, C-CR are the column effects model which gives the effects of the column, RC and the residuals of the models.

TABLE 9: THE ANOAS SHOWING DIFFERENCES BETWEEN THE MODELS

	Effects	Models	Df	G^2	Percentage
1.	General	O-U	1	19868.2138	17.13%
2.	Rows	U-C	7	82893.9941	68.17%
3.	Column-effects which provides the Row-effects	C-RC	9	1615.2791	1.23%
4.	Residual	RC	63	153277.3587	13.47%
	Total	O	80	123960.12346	100.00%

From the ANOAS table we created, the uniform association model (U) as shown in table 7 above, covers 17.13% of the total baseline chi-squared X^2 distribution, the Row model (R) accounts for a large percentage of 68.17%, the Column-effects which gives the Row-effects (C-RC)) covers a small percentage of 1.23%. Finally, the row column effects (Residuals) are weak also because the RC model accounts for only 13.47% of the baseline chi-squared value.

We could therefore observe that at the rate of 13.47%, the variation which is attributed to the null-independence has been measured from the model of RC. This rate is quite satisfactory and we can say that the corresponding percentage of population change in 25 countries of the European Union (Belgium, Czech Republic, Denmark, Germany, Estonia, Ireland, Greece, Spain, France, Italy, Cyprus, Latvia, Lithuania, Luxembourg, Hungary, Malta, Netherlands, Austria, Poland, Portugal, Slovenia, Slovakia, Finland, Sweden and United Kingdom), as shown from the data depended slightly positive on the association of both the countries and also from the years under our study (2003-2017).

We could as well say that the percentage of the data is explained by the column effects model (on the local odds ratios in a multiplicative way), thus, giving it a satisfactory fit but not acceptable due to poor adjustment that was found earlier in our study, i.e. the value of the Pearson chi-squared X² distribution for the 95% reference pointare much smaller for the model (RC).

Furthermore, because our best model (RC) under our study is unacceptable because it has a bad fit based on the index BIC upon clear examination to test the accuracy, quality or satisfactory fit of the model, therefore, we shall proceed to examining the multivariate model to find the model with a satisfactory fit.

III. THE MULTIVARIATE MODELS

In the RC (M) association model, M represents the dimension fit to be, which is utilized by the row-column dimension (RCDIM PROGRAM). As shown in Table 10 below the multivariate model RC (M=4) is the acceptable model with the best fit [13].

The results are as follows:

TABLE 10: THE MULTIVARIATE MODE

	TABLE 10: THE MICETIVARIATE MODEL								
Ξ	Models	RC(1)	RC(2)	RC(3)	RC(4)				
	G^2	15327.85142	486.20303	233.01260	22.35796				
	X^2	82.6494	65.2594	49.8718	36.35796				
	d.f.	63	48	35	24				
	D	0.00093	0.00017	0.00011	0.00003				

Model RC (1) multivariate row, column, M=1 Model RC (2) multivariate row, column, M=2 Model RC (3) multivariate row, column, M=3 Model RC (4) multivariate row, column, M=4

A. Examination of the multivariate model

In the RC (M) association model, M represents the dimension to be fit, which is utilized by PROG RCDIM. As it is shown above the multivariate model RC (M=4) is the acceptable model with the best fit.

The multivariate model RC (1) with M=1 has likelihoodratio chi-square statistic $G^2 = 15327.85142$ with 63 degrees of freedom (d.f.). Thus, we find out that the 95% reference point of the X² distribution is 82.6494. It has a bad fit. We could conclude therefore that this model has a bad fit because the X² distribution is much smaller than the likelihood-ratio chi-square statistic G².

Moreover, the multivariate model RC (2) with M=2 has likelihood-ratio chi-square statistic $G^2 = 486.20303$ with 48 degrees of freedom (d.f.). The 95th percentile of the reference chi-square distribution is 65.2594. Similarly, this model has a bad fit since the X^2 distribution.

The multivariate model RC (3) with M=3, has likelihoodratio chi-square statistic $G^2 = 22.35796$ with 35 degrees of freedom (d.f.). Thus, we find out that the 95% reference point of the X^2 distribution is 49.8718. It has a very bad fit, thus, making it unacceptable.

Finally, the multivariate model RC (4) with M=4, has likelihood-ratio chi-square statistic $G^2 = 22.35796$ with 24 degrees of freedom (d.f.). Thus, we find out that the 95% reference point of the X² distribution is 36.35796. Here we could see that the multivariable RC model with M = 4 has the best fit. We equally observed that the model M=4 covers $\{(165674.62855-62.91899) / 165674.62855\} = 99.9\% \text{ of all }$

When a model with a smaller M has is satisfactory fit, it is accepted because it gives a better explanation of the interaction between the rows and columns. Therefore, we'll prefer the multivariate model M=4 which has a perfect fit.

B. Evaluation of the multivariate model

The practical implementation of multivariate statistics to a particular problem involve several types of univariate and multivariate analysis in order to understand the relationships between variables and their relevance to the actual problem being studied. In addition, multivariate statistics is concerned with multivariate probability distributions, in terms of both: how these can be used to represent the distributions of observed data; and how they can be used as part of statistical inference, particularly where several different quantities are of interest to the same analysis [13].

The expected frequencies under the independent and column effects models for population change in EU25: absolute numbers and crude rates from 2003-2017 are given below:

Note: The multivariate model RC (M=4) seems to give much better fit, particularly at the end of nominal scale.

							0714	7.70	0520
					8	9	9714	7579	9730
TA	ABLE 11: EVA	LUATION	OF THE MULTIVA	RIATE MODEL	8	10	10627	7286	10599
	••	-	Values of 0	Values of RC (M=4)	8	11	11317	7417	11221
Countries	Years	Data	Model (f _{ij} ¹)	Model (F _{ij} ²)	8	12	11319	7564	11411
ROW	COLUMN	DATA	OBSERVED	EXPECTED	9	1	23163	40956	23199
1	1	95	153	80	9	2	23917	23146	23856
1	2	85	86	106	9	3	31370	48218	31341
	2			98	9	4	31819	26365	31795
1	3	85	180		9	5	30904	25808	31100
1	4	160	98	141	9	6	30892	25567	30780
1	5	187	96	172	9	7	25473	23480	25555
1	6	183	95	195	9	8	26820	24788	26744
1	7	183	88	166	9	9	29332	24197	29332
1	8	126	93	146	9	10	21937	23262	21922
1	9	74	90	75			29489		
1	10	41	87	39	9	11		23680	29420
1	11	13	88	15	9	12	28498	24148	28571
1	12	13	90	11	10	1	28553	37790	28460
2	1	473	600	462	10	2	28724	21356	28850
2	2	544	339	556	10	3	26785	44490	26857
2	3	425	707	437	10	4	26486	24327	26490
2	4	778	386	733	10	5	26680	23812	26292
2 2					10	6	25166	23590	25428
2	5	365	378	421	10	7	25177	21665	25119
2	6	294	375	269	10	8	23962	22872	24049
2	7	231	344	234	10	9	24267	22326	24285
2	8	447	363	452	10	10	24516	21464	24497
2	9	249	355	252	10	11	23761	21849	23663
2	10	315	341	305	10	12	23745	22281	23833
2	11	326	347	301					
2	12	443	354	469	11	1	18937	27856	18978
3	1	1820	2835	1804	11	2	19572	15742	19515
3	2	1756	1602	1776	11	3	20863	32794	20826
3	3	1723	3337	1739	11	4	21037	17932	21084
3	5	1947	1786	1967	11	5	21653	17553	21709
3	6	2009	1770	2004	11	6	21833	17389	21789
3	7	1921	1625	1913	11	7	18429	15970	18474
3					11	8	19188	16859	19134
	8	1967	1716	1982	11	9	11822	16457	11817
3	9	1938	1675	1941	11	10	18110	15821	18117
3	10	2045	1610	2037	13	2	35	41	33
3	11	2043	1639	2026	13	3	43	85	42
3	12	2045	1671	2063	13	4	47	46	46
4	1	4192	5678	4230	13	5	33	45	43
4	2	3970	3209	3923	13	6	46	45	40
5	8	7428	5419	7424					
5	9	5723	5289	5723	13	7	43	41	44
5	10	4469	5085	4477	13	8	64	44	63
5	11	3768	5177	3768	13	9	55	43	55
5	12	4500	5279	4497	13	10	54	41	54
6	1	27	57	30	13	11	57	42	60
6	2	26	32	22	13	12	73	43	70
6	3	26	67	23	14	1	154	303	164
6	4	20	36	21	14	2	152	171	142
6	5	23	36	32	14	3	152	356	143
6	6	47	35	41	14	4	165	195	187
			33	20	14	5	200	191	195
6	7	26	32	29	14	6	200	189	195
6	8	37	34	33	14	8	236	183	238
6	9	25	33	25	14	9	270	179	266
6	10	56	32	56	14	10	201	172	211
6	11	70	33	70	14	11	223	175	262
6	12	78	33	78	14	12	338	178	299
7	1	3493	7603	3572	15	1	808	1792	807
7	2	3685	4297	3588	15	2		1013	939
7	3	4238	8951	4174	15	2	933		
7	4	4386	4895	4438	15	3	1022	2110	1024
7	5	5125	4791	5333	15	4	1195	1154	1198
7	6	6094	4746	5932	15	5	1289	1129	1281
7	7	6257	4359	6267	15	6	1306	1119	1305
7	8	6252	4602	6205	15	7	1157	1027	1133
7	9	5836	4492	5816	15	8	1187	1085	1205
7	10	6005	4318	6040	22	2	92	102	95
7	11	5312	4396	5463	22	3	91	211	93
7	11		4396	5105	22	4	121	116	115
		5250			22	5	118	113	123
8	1	3985	12828	3902	22	6	126	112	124
8	2	4884	7250	4991	22	7	129	103	127
8	3	5993	15103	6064	22	8	135	109	137
8	4	8427	8258	8351	22	9	157	106	157
8	5	9542	8083	9376	22	10	135	100	134
8	6	9751	8008	9882	22	11	137	102	136
8	7	8793	7355	8751	22				
8	8	10143	7764	10216		12	135	106	136
		_	_		23	1	95	146	97

23	2	125	83	121
23	3	65	172	63
23	4	87	94	88
23	5	89	92	96
23	6	100	91	96
23	7	83	84	88
23	8	88	89	83
23	9	118	86	118
23	10	96	83	96
23	11	126	85	123
23	12	120	86	123
24	1	1766	2179	1731
24	2	1643	1231	1693
24	3	1602	2565	1636
24	4	1545	1403	1478
24	5	1540	1373	1555
24	6	1574	1360	1572
24	7	1513	1249	1464
24	8	1256	1319	1311
24	9	1282	1287	1284
24	10	1436	1238	1431
24	11	1289	1260	1310
24	12	1303	1285	1283
25	1	824	917	816
25	2	668	518	665
25	3	548	1080	558
25	4	600	590	534
25	5	483	578	595
25	6	677	573	630
25	7	562	526	613
25	8	633	555	602

¹f_{ii:} Expected frequencies of the independence model.

From Table 11 above, it is evident that the prices (value) of the model RC (M = 4) fully fitted to the data.

IV. LOGARITHMS OF RC (M = 4) MODEL

The RC (M=4) from the RCDIM (ROW TAU-S) are as follows:

```
Belgium: \hat{\tau}_1 = \ln(-0.330335) = -1.107647988
Czech Repub.:\hat{\tau}_2 = -\ln(0.99118) = -0.00904
Denmark: \hat{\tau}_3 = \ln(-0.986963) = -0.01312272759
Germany: \hat{\tau}_4 = \ln(1.685072) = 0.52180
Estonia: \hat{\tau}_5 = \ln(-3.44861) = -1.2380
Ireland: \hat{\tau}_6 = \ln(-1.335252) = -0.2891200381
Greece: \hat{\tau}_7 = \ln(-0.275362) = -1.289668683
Spain: \hat{\tau}_8 = \ln(1.034778) = 0.03418691096
France: \hat{\tau}_9 = \ln(1.415027) = 0.3471486122
Italy:\hat{\tau}_{10} = \ln (1.388701) = 0.3283687778
Cyprus:\hat{\tau}_{11} = \ln(-1.82367) = -0.601
Latvia: \hat{\tau}_{12} = \ln(-3.130036) = -1.1410
Lithuania: \hat{\tau}_{13} = \ln(-1.700391) = -0.5309
Hungary: \hat{\tau}_{14} = \ln(0.08966) = -2.4117
Malta: \hat{\tau}_{15} = \ln (-1.82028) = -0.5990
Netherlands: \hat{\tau}_{16} = \ln(1.950344) = 0.6680
Austria: \hat{\tau}_{17} = \ln(-0.575237) = -0.5529731492
Poland: \hat{\tau}_{18} = \ln(1.121537) = 0.115
Portugal: \hat{\tau}_{19} = \ln(-0.335356) = -1.092562625
Romania: \hat{\tau}_{20} = \ln(-0.1247) = 2.0818
Slovenia: \hat{\tau}_{21} = \ln(-2,2101) = -0.7930
Slovakia: \hat{\tau}_{22} = \ln(-2,41695) = -0.8825
Finland: \hat{\tau}_{23} = \ln(0.2956) = -1.2187
Sweden: \hat{\tau}_{24} = \ln(-0.56996) = 0.5622
```

V. RESEARCH FINDINGS: COMPARISON OF THE POPULATION CHANGE IN EU25

By comparing some of the countries in EU25 to ascertain the variation within the population from 2003-2017, it

For example, we compared France and Greece and observed that $\hat{\tau}_9 - \hat{\tau}_7 = 1.296$, exp (1.296) = 3.61. This means that the variation within the population in France is more than three times compared to that of Greece.

In the case of Italy and the Netherlands, we find that τ^{10} - $\tau^{16} = 0.384$, exp (0.384) = 2.421. That is, the population change in Italy outnumbered that of Holland by 2.42 proportionally.

The difference between Denmark and Ireland was slightly higher, where τ^3 - $\tau^6 = 0.0131521111 - 0.289098319 =$ 0.30225043, exp (0.30225043) = 1.35. In other words, the population change in Denmark increased slightly with 1.35 than Ireland.

As for the difference between Austria and Portugal, we observe that τ^{17} - $\tau^{19} = 0.553021826$ - 1.0926491 = -0.539627274, exp (-0.539627274) = 0.58. That is, the number of inhabitants (variation within the population change) is 0.583 lower than that of Portugal.

In the case of Mediterranean countries like Greece and Spain, we see that $\hat{\tau}_7 - \hat{\tau}_8 = 1.289668683 - 0.03418691086 =$ 1.323855594, exp (1.323855594) = 0.27. We could say that Greece has a lower population change by 0.27 than Spain.

By comparing Belgium and Greece, we observed that, $\hat{\tau}_1$ - $\hat{\tau}_7 = 1.107647988 - 1.289668683$, exp (1.124589) = -0.83. It means that Belgium has a lower population change by 0.83 than Greece.

Finally, in the Scandinavian countries such as Finland and Sweden, the difference between them is, $\hat{\tau}_{23}$ - $\hat{\tau}_{24}$ = -1.2187-0.5622 = -1.7809 with exp (-1.7809) = 0.168, thus, the variation within the population in Finland is smaller than its neighboring country Sweden.

VI. SUMMARY

The Highest proportion of government expenditure in the EU goes to social protection and health. In 2017, total government expenditure in the European Union (EU) amounted to 46.7% of gross domestic product (GDP). This share has steadily decreased since 2012, when it stood at 49.7% of GDP.

Among the main functions of general government expenditure in the EU, 'social protection' is by far the most important, equivalent to 19.2% of GDP in 2017. The next most important areas are 'health' (7.0%), 'general public services' (6.0%) such as external affairs and public debt transactions, 'education' (4.6%) and 'economic affairs' (4.4%). The functions 'public order and safety' (1.7%), 'defence' (1.2%), 'recreation, culture and religion' (1.1%), 'environmental protection' (0.8%) and 'housing and community amenities' (0.6%) have more limited weights.

Highest natural growth rates were recorded in Ireland, Cyprus, France, Luxembourg and the United Kingdom. In 2010, 5.4 million children were born in the EU27. The crude birth rate was 10.7 per 1000 inhabitants, the same as in 2009. The highest birth rates were recorded in Ireland

 $UK:\hat{\tau}_{25} = \ln(3.06409) = 1.1197$

 $^{^2\!}F_{ij}\!$ Expected frequencies of the multivariate model RC (M = 4).

(16.5%), the United Kingdom (13.0%), France (12.8%), Cyprus (12.4%) and Sweden (12.3%), and the lowest rates in Germany (8.3%), Latvia (8.6%), Hungary (9.0%), Italy (9.3%), Austria (9.4%), Portugal (9.5%) and Malta (9.6%).

There were 4.8 million deaths registered in the EU27 in 2010. The crude death rate was 9.7 per 1000 inhabitants, unchanged compared with 2009. The highest death rates were observed in Bulgaria (14.6%), Latvia (13.4%), Hungary (13.0%), Lithuania (12.8%) and Romania (12.1%), and the lowest rates in Ireland (6.2%), Cyprus (6.7%), Malta (7.2%) and Luxembourg (7.4%).

Consequently, the highest natural growth of the population (the difference between live births and deaths per 1000 inhabitants) was registered in Ireland (+10.3%), well ahead of Cyprus (+5.7%), France (+4.4%), Luxembourg (+4.2%) and the United Kingdom (+3.9%). Eight Member States had a negative natural growth, with the largest declines in Latvia (-4.8%), Bulgaria (-4.6%), Hungary (-4.0%), Germany and Romania (both -2.2%). Highest population growth was recorded in 2017 in Luxembourg, Sweden, Malta, Belgium and the United Kingdom.

In 2010, over 60% of the increase in the EU27 population came from migration. In relative terms, Luxembourg (+15.1%), Malta (+5.4%), Sweden (+5.3%), Italy (+5.2%) and Belgium (+5.1%) had the largest net inflows, while Lithuania3 (-23.7%) and Ireland (-7.5%) recorded the highest net outflows.

In conclusion, the population increased in twenty Member States and decreased in seven, with considerable variations between Member States. The largest relative increases were observed in Luxembourg (+19.3%), Sweden (+8.0%), Malta (+7.8%), Belgium (+7.2%) and the United Kingdom (+6.6%), and the largest decreases in Lithuania3 (-25.7%), Latvia (-8.4%) and Bulgaria (-7.8%).

In the context of the annual demographic balance, Eurostat produces net migration figures by taking the difference between total population change and natural change; this concept is referred to as net migration plus statistical adjustment.

Most of the global increase is attributable to a small number of countries. From 2017 to 2050, it is expected that half of the world's population growth will be concentrated in just nine countries: India, Nigeria, the Democratic Republic of the Congo, Pakistan, Ethiopia, the United Republic of Tanzania, the United States of America, Uganda and Indonesia (ordered by their expected contribution to total growth).

The group of 47 least developed countries (LDCs) continues to have a relatively high level of fertility, which stood at 4.3 births per woman in 2010-2015. As a result, the population of these countries has been growing rapidly, at around 2.4% per year. Although this rate of increase is expected to slow significantly over the coming decades, the combined population of the LDCs, roughly one billion in 2017, is projected to increase by 33% between 2017 and 2030, and to reach 1.9 billion persons in 2050.

Similarly, Africa continues to experience high rates of population growth. Between 2017 and 2050, the populations of 26 African countries are projected to expand to at least double their current size.

The concentration of global population growth in the

poorest countries presents a considerable challenge to governments in implementing the 2030 Agenda for Sustainable Development, which seeks to end poverty and hunger, expand and update health and education systems, achieve gender equality and women's empowerment, reduce inequality and ensure that no one is left behind.

VII. CONCLUSION

All the six association models show bad fit. Therefore, we proceeded to the multivariate model in which the row column-effects RC multivariate model of RC ((M = 4) is better than all because it covers 99.98% of the data, thus, giving the best fit among all. However, to be more precise, the population of a country is affected or influenced by several factors; these could be as a result of:

- The standard of living of each country; every country can prosper if its citizens are productive and do not display behaviors that hinder development such as corruption and poor work ethic [16].
- The degree of utilization of the workforce, which is directly related to efficiency in the production of goods and services [17].
- A country's mortality rate and
- Several other factors which are difficult to be determined in each country

We should mention that Θ , which is 0.08906, i.e. approximately equal to 1, show that we have independence between the variables. To determine the degree of correlation (association) between these countries and years, we use the θ of the second model, (the uniform association model which is typified by (U) for calculation. More specifically, the innate correlation index in log-linear form is equal to:

$$\log(F_{ij}) = \lambda + \lambda_{A(i)} + \lambda_{B(j)} + \varphi \chi_i y_j$$

where φ is a single parameter for interaction. The parameter of interaction

$$\varphi = \log \theta = \log (0.08906)$$

$$\Phi = \ln (0.08906) = -2.41844498$$

$$|\varphi 1/2| = \sqrt{-2.41844498} = 1.555135036$$

$$\varphi 1/2 = \sqrt{1.555135036} = 1.24705053$$

Since there is a correlation based on our study on population change in EU25: absolute numbers and crude rates, the general observation is that in all the 25 countries of the European Union, over the years from 2003–2017, there was a slight increase in each year of their population.

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